

Math 315-003
April 2,3,5, 2004
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Test 3
Show relevant work!

Name_____

1. State and prove the First Fundamental Theorem of Calculus.

2. Suppose the function $f : \mathbf{R} \rightarrow \mathbf{R}$ is continuous. Define

$$G(x) = \int_0^x (x-t)f(t)dt \quad \text{for all } x.$$

Prove that $G''(x) = f(x)$ for all x .

3. Describe in words the n^{th} degree Taylor Polynomial for a function f at the point x_0 . Explain why it is uniquely determined.

4. Show the number e is irrational

5. Show that the Taylor expansion of $f(x) = \sin(x)$ at $x_0 = 0$ converges for all points x .

6. Suppose that $\sum_{k=1}^{\infty} a_k$ converges. Show $\lim_{n \rightarrow \infty} a_n = 0$.

7. Define what it means for a sequence to be Cauchy and show that a convergent sequence is Cauchy.

8. Suppose $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ are series of positive numbers such that

$\lim_{k \rightarrow \infty} \left(\frac{a_k}{b_k} \right) = \lambda$ and $\lambda > 0$. Show that $\sum_{k=1}^{\infty} a_k$ converges if and only if the series

$\sum_{k=1}^{\infty} b_k$ converges.

9. For a number r such that $|r| < 1$, show $\sum_{k=1}^{\infty} r^k$ converges.

10. Does the series $\sum_{k=1}^{\infty} \frac{1}{(k+1)\ln(k+1)}$ converge? Prove your assertion.